

Anharmonic Elasticity of Smectics A and the Kardar-Parisi-Zhang Model

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We relate anharmonic *equilibrium* thermal fluctuations of smectics A to fluctuations of the Kardar-Parisi-Zhang (KPZ) *dynamical* model for a growing interface. The KPZ model in 1+1 dimensions is one to one related to a 2D smectic elastic model whose scaling behavior is then obtained *exactly*. The KPZ model in 2+1 dimensions maps into an elastic critical point of 3D smectics A with *broken inversion symmetry* (head-to-tail packing of layers). We discuss the elasticity and fluctuations of these novel smectic- A phases.

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One of the most striking phenomena in the statistical physics of elastic media is the breakdown of the harmonic (Hookean) elasticity in smectics A [1]. Grinstein and Pelcovits demonstrated that anharmonic effects of thermal fluctuations cause, at long length scales, a nontrivial logarithmic renormalization of 3D smectic elastic constants inducing, in particular, a breakdown of the classical elasticity theory: linear Hooke's law is replaced by a nonlinear strain response to *arbitrarily* weak external stresses. This seminal discovery inspired numerous studies in liquid crystals, spin glasses, exotic magnets, membranes, and nematic polymers [2]. These phenomena exist in seemingly remote physical problems such as Rayleigh-Bénard roll instability [3] (analog of 2D smectics [4]) and pion condensation in neutron stars (analog of 3D smectics [5]).

On the other side, similar breakdown of the classical, harmonic fluctuation theory is the main theme also of the statistical physics of growing interfaces [6]. This field is actively being developed, in particular, due to a direct interest in scaling properties of interfaces growing in the presence of external fluxes [7], as well as its relationship to other physical problems [6,8].

This Letter reveals a deep relationship between *non-equilibrium* statistical physics of growing interfaces and *equilibrium* statistical physics of smectics A . We relate the Kardar-Parisi-Zhang model [7] for the dynamics of $(d-1)$ -dimensional growing interface to *nonlinear* elasticity theory of d -dimensional smectics A . For 2D smectics [4], we obtain the *exact* scaling behavior of the nonlinear elasticity theory, with correlations of smectic layer displacements u of the form $\langle [u(x,z) - u(0,0)]^2 \rangle^{1/2} = K(x,z)$, with $K(x,0) \sim x^{1/2}$, and $K(0,z) \sim z^{1/3}$ [z axis (x axis) is normal (parallel) to the layers]. Linear Hooke's law breaks down and is replaced by the nonlinear law of strain \sim (stress)^{2/3}, for *arbitrarily weak* external stresses normal to layers. These results hold for *stricto sensu* elastic models with no dislocations, which are actually free in a 2D smectic and convert it, at any finite temperature T , into a nematic phase at length scales longer than the sizes of cybotactic groups ξ_x and ξ_z ($\xi_x, \xi_z \rightarrow \infty$ as $T \rightarrow 0$) [4]. Our theory enables us to obtain the exact

form of the *anisotropic scaling* at the zero- T transition from the 2D smectic to the nematic phase: $\xi_z \sim \xi_x^{3/2}$. All these results are of interest not only for 2D smectics, but also for their analogs such as Rayleigh-Bénard systems [3], or stripe domain phases in thin ferromagnetic films in which the anisotropic scaling has also been observed recently [9].

Another important contribution of this work is the elastic model for smectics A with *broken spatial inversion symmetry* [10,11], such as ferroelectric smectics A with an average dipolar moment normal to the layers, resembling a lamellar phase of surfactant monolayers stacked according to the "head-to-tail" rule [10]. Numerous molecular architectures capable of forming such phases have been proposed in the past [11]. Their experimental realization is, however, of a quite recent date [12]. As discussed here, the elastic model of these phases must contain a new, rotationally invariant term of the form eH , coupling the strain e and the layer curvature H . This term is forbidden in ordinary, inversion symmetric smectics A since it changes sign under spatial inversion. We find that the KPZ model in 2+1 dimensions maps into a novel *elastic critical point* of 3D smectics A with broken inversion symmetry. At this point, displacement fluctuations strongly diverge as *power laws* of the system sizes L_x and L_z : for $L_z \gg L_x$, $\langle u^2 \rangle^{1/2} \sim L_x^\alpha$, $\alpha \approx 0.4$; for $L_x \gg L_z$, $\langle u^2 \rangle^{1/2} \sim L_z^\beta$, $\beta \approx 0.25$, in contrast to a much weaker Landau-Peierls logarithmic divergence in inversion symmetric smectics [13].

We first propose a nonlinear elastic model for smectics A with broken inversion symmetry and screened long-range dipolar interactions. Their elastic energy can be represented via a sum of layers' energies, $E = \sum_n E_n = \int (dz/l) E_{z/l}$, with l the equilibrium layer thickness. E_n contains a compressional, $E_{n,\text{com}}$, and a curvature contribution, $E_{n,\text{curv}}$. $E_{n,\text{com}}$ is generally of the form $E_{n,\text{com}} = \frac{1}{2} B \int dS e^2$, where the integral is over the area of the n th layer, specified by its height $h(\mathbf{x}, z)|_{z=nl}$, $dS = d^d \mathbf{x} [1 + (\nabla_{\mathbf{x}} h)^2]^{1/2}$, and $e = \partial_z h [1 + (\nabla_{\mathbf{x}} h)^2]^{-1/2} - 1$ is the Lagrangian strain [the displacement $u(\mathbf{x}, z) = h(\mathbf{x}, z) - z$ changes layer thickness, measured normal to the layer, by $\delta l = el$]. Layers are fluid, and $E_{n,\text{curv}}$ is,

as for fluid membranes [14], $E_{n,\text{curv}} = \frac{1}{2} K \int dS (H^2 + 2H_0 H)$, with H , the layer curvature, $H = \nabla_{\mathbf{x}} \{ \nabla_{\mathbf{x}} h / [1 + (\nabla_{\mathbf{x}} h)^2]^{1/2} \}$, while the constant H_0 is the so-called *spontaneous curvature*. Its presence *breaks the inversion symmetry* $(h, \mathbf{x}) \rightarrow (-h, -\mathbf{x})$. $H_0 \neq 0$ can arise only if nematogens do not have a center of inversion (head-and-tail molecules) *and* if most of the tails are on one while

most of the heads are on the other layer side. Such a layer *alone* would tend to bend towards one, say tail, side. Now, let us stack the layers so that the head side of a layer is adjacent to the tail side of its neighbor (head-to-tail rule [10]). By summing layers' energies, we obtain the *full* nonlinear, rotationally invariant smectic elastic Hamiltonian

$$E_{\text{sm}} = \int dz d^{d-1} \mathbf{x} \{ 1 + (\nabla_{\mathbf{x}} h)^2 \}^{1/2} \left[\frac{B_{\text{sm}}}{2} e^2 + \frac{K_{\text{sm}}}{2} H^2 + \gamma_{\text{sm}} H \right] \\ = \int dz d^{d-1} \mathbf{x} \{ 1 + (\nabla_{\mathbf{x}} h)^2 \}^{1/2} \left[\frac{B_{\text{sm}}}{2} e^2 + \frac{K_{\text{sm}}}{2} H^2 - \gamma_{\text{sm}} e H \right]. \quad (1)$$

Here $B_{\text{sm}} = B/l$, $K_{\text{sm}} = K/l$, and $\gamma_{\text{sm}} = KH_0/l$. The two forms of H_{sm} in Eq. (1) differ by a surface term not affecting bulk fluctuations. The eH terms in (1), with $\gamma_{\text{sm}} \sim H_0$, is *odd* under spatial inversion $h(\mathbf{x}, z) \rightarrow -h(-\mathbf{x}, -z)$ under which $e \rightarrow e$, $H \rightarrow -H$. $\gamma_{\text{sm}} = 0$ in ordinary, symmetric smectics A because either the heads (tails) are equally distributed between layer sides (so $H_0 = 0$), or, if this is not the case, asymmetric layers are arranged head to head, tail to tail, so that spontaneous curvature contributions of neighboring layers cancel.

In 2D (and *only* in 2D) the γ_{sm} term in (1) becomes a derivative contributing only to the boundary energy: Note that the spontaneous curvature of each layer contributes the energy $\sim \int dx [1 + (\partial_x h)^2]^{1/2} H = \int ds \partial \theta / \partial s$, with s the arclength, and θ the local layer tilt angle $[= \tan^{-1}(\partial_x h)]$. Thus, the inversion-symmetry-breaking term does not affect bulk fluctuations in 2D. From the point of view of bulk phonons, the ordinary ($\gamma_{\text{sm}} = 0$) and asymmetric smectics A ($\gamma_{\text{sm}} \neq 0$) are *identical* in 2D.

We now relate the nonlinear smectic Hamiltonian (1) to the KPZ model for the evolution of the profile $h(\mathbf{x}, t)$ of a $(d-1)$ -dimensional growing interface [7]. The full nonlinear, rotationally invariant dynamical model for the interface of an isotropic (amorphous) cluster growing in an isotropic depositing flux has the form

$$\frac{\partial h}{\partial t} = [1 + (\nabla_{\mathbf{x}} h)^2]^{1/2} (\lambda + \nu H) + [1 + (\nabla_{\mathbf{x}} h)^2]^{1/4} \eta(\mathbf{x}, t). \quad (2)$$

λ is the (bare) mean velocity of the interface. In the following, we chose a time unit such that $\lambda = 1$. ν is a surface tension, and η is a noise, with the distribution

$$P(\eta) \sim \exp \left[-\frac{1}{4D} \int dt d^{d-1} \mathbf{x} \eta^2(\mathbf{x}, t) \right]. \quad (3)$$

To relate (2) and (3) to the smectic model (1), we first identify the *time* coordinate t of the KPZ model with the smectic *spatial* coordinate z , $t = z$. Next, we map the *dynamical* problem (2) and (3) in $d-1$ spatial dimensions (\mathbf{x}) into an *equilibrium* statistical mechanics problem for the field $h(\mathbf{x}, z)$ in d spatial dimensions (\mathbf{x}, z).

This can be accomplished by applying the classical dynamics path integral formalism [15], yielding the probability weight $P(h)$ of a field configuration h . $h(\mathbf{x}, z)$ can be interpreted now as the smectic layer height function. (So, we identify the sequence of snapshots of the KPZ interfaces taken at equal time intervals with a stack of smectic layers.) $P(h)$ is simply obtained from $P(\eta)$ in (3), by changing variable $\eta \rightarrow h$. By (2), with $\lambda = 1$, $\eta(h) = [1 + (\nabla_{\mathbf{x}} h)^2]^{1/4} (e - \nu H)$, where e and H are smectic local strain and layer curvature. Thus, $P(h) = P(\eta) \times J(h)$, with $J(h) = |D\eta/Dh|$. The Jacobian J can be ignored without qualitative consequences for the following [16]. Thus, eventually, $P(h)$ assumes the form of a Boltzmann factor, $P(h) \sim \exp[-H_{\text{eff}}(h)]$, with the effective Hamiltonian

$$H_{\text{eff}}(h) = \frac{1}{4D} \int dz d^{d-1} \mathbf{x} \{ 1 + (\nabla_{\mathbf{x}} h)^2 \}^{1/2} [e - \nu H]^2. \quad (4)$$

Note that $H_{\text{eff}}(h)$ is equivalent to the smectic Hamiltonian (1) for a special choice of B_{sm} , K_{sm} , and γ_{sm} ensuring that the expression in square brackets in the second line of Eq. (1) is, as in Eq. (4), a *full square*. It follows that for the special value of γ_{sm} ,

$$\gamma_{\text{sm}} = \gamma_c = \pm (K_{\text{sm}} B_{\text{sm}})^{1/2}, \quad (5)$$

the *equilibrium* behavior of the smectic elastic model (1) is directly related to the dynamical *behavior* of the KPZ model (2) and (3) in *any* d .

Recall now that, for $d=2$, the γ_{sm} term in (1) contributes only to the boundary energy. Thus, in 2D, Eq. (4) reduces to the ordinary smectic Hamiltonian, Eq. (1) with $\gamma_{\text{sm}} = 0$, $B_{\text{sm}} = 1/2D$, and $K_{\text{sm}} = \nu^2/2D$. So, the standard 2D smectic elastic model is *one to one* related to the KPZ model. Using this and the *exact* results for the KPZ model in 1+1 dimensions [7], we arrive at the following conclusions about 2D smectics: (I) Correlations of smectic displacements, $u(x, z) = h(x, z) - z$, are given by $\langle [u(x, z) - u(0, 0)]^2 \rangle^{1/2} = K(x, z)$, with

$$K(x, z) = |x|^a \phi(|z|/|x|^{a/\beta}), \quad (6)$$

where $\phi(s) \rightarrow s^\beta$ for $s \rightarrow \infty$, while $\phi(s) \rightarrow \text{const}$ for

$s \rightarrow 0$. [Thus, $K(x,0) \sim |x|^\alpha$, $K(0,z) \sim |z|^\beta$.] Here $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ exactly [7]. (II) Smectic elastic constants undergo a nontrivial renormalization at long length scales (small wave vectors q) of the form $K_{sm}(q) \sim |q_x|^{-1/2}$, for $q_z=0$, and $K_{sm} \sim |q_z|^{-1/3}$, for $q_x=0$, while $B_{sm}(q) \sim |q_x|^{1/2}$, for $q_z=0$, and $B_{sm} \sim |q_z|^{1/3}$, for $q_x=0$ [17]. This renormalization causes a breakdown of linear elasticity theory: The strain responds *nonlinearly* to *weak* external stresses S along the z direction [18],

$$\langle e \rangle \sim S^{2/3}.$$

(III) The above results hold for 2D smectics with purely phonon-type excitations. Behavior at long length scales is, however, affected by dislocations which are free in 2D smectics at any finite temperature T [4]. They convert a 2D smectic A , at large length scales, into a nematic, so that the smectic order still persists inside anisotropic domains (cybotactic groups) with sizes ξ_z and ξ_x along z and x directions: $\xi_x \xi_z = \xi_D^2 = 1/n_D$, where $n_D \sim \exp(-\text{const}/T)$ is the density of free dislocations. Harmonic elasticity theory, with $\xi_z \sim \xi_x^2$, then yields $\xi_x \sim \xi_D^{2/3}$ and $\xi_z \sim \xi_D^{4/3}$, as found by Toner and Nelson [4]. The above anharmonic effects produce a different scaling for large enough cybotactic groups. According to Eq. (6), $\xi_z \sim \xi_x^{a/\beta} = \xi_x^{3/2}$, and thus $\xi_x \sim \xi_D^{4/5}$ and $\xi_z \sim \xi_D^{6/5}$.

We now turn to 3D smectics A with broken inversion symmetry [10–12], by first discussing their behavior for the *special* value of the symmetry-breaking coupling $\gamma_{sm} = \gamma_c = \pm (K_{sm} B_{sm})^{1/2}$, Eq. (5), for which they are related to the KPZ model in 2+1 dimensions. By using this relationship, we conclude the following for $\gamma_{sm} = \gamma_c$: (I) The displacement correlations have the form of Eq. (6), with $\beta = a/(2-a)$ (in any d [7]). For $d=3$, $a \approx 0.40$ and $\beta \approx 0.25$ [19]. (II) For $\gamma_{sm} = \gamma_c$, elastic constants of 3D smectics A undergo a nontrivial renormalization at small wave vectors of the form ($d=3$) $K_{sm} \sim q_x^a$, for $q_z=0$, and $K_{sm} \sim q_z^{a/(2-a)}$, for $q_x=0$, whereas $B_{sm}(q) \sim [K_{sm}(q)]^3$ [17]. So, both K_{sm} and B_{sm} vanish at long length scales ($q \rightarrow 0$). (III) This softening of elastic constants causes a breakdown of Hooke's law: A weak stress S normal to layers produces a strain $\langle e \rangle \sim S^{\eta_S}$, with $\eta_S = 2(1-a)/(d-1+a)$ [18]. This, with $a \approx 0.4$ in $d=3$ [19], gives $\eta_S \approx 0.5$, i.e., $\langle e \rangle \sim S^{1/2}$. (IV) The softening of elastic constants produces violent displacement fluctuations diverging for $\gamma_{sm} = \gamma_c$ as *power laws* of the system sizes: $\langle u^2 \rangle^{1/2} \sim L_x^a$ for $L_z \gg L_x$, and $\langle u^2 \rangle^{1/2} \sim L_z^\beta$ for $L_x \gg L_z$. This divergence is much stronger than the well-known Landau-Peierls logarithmic divergence in ordinary smectics A having $\gamma_{sm}=0$ [13]. For $\gamma_{sm} = \gamma_c$, strong displacement fluctuations destroy long-range translational order and produce *exponentially* decaying translational correlations. This is in marked contrast to the situation for $\gamma_{sm}=0$ with a power-law decay of translational correlations [13]. The state at $\gamma_{sm} = \gamma_c$ would appear like a nematic. Nonetheless, thermal undulations dephasing translational correlations do not destroy

the integrity of smectic layers which only assume a rough appearance similar to that of successive snapshots of the KPZ model interfaces [6]. Correlations of director fluctuations $\langle \nabla_x h(\mathbf{x}, z) \nabla_x h(\mathbf{0}, 0) \rangle$ decay, for $\gamma_{sm} = \gamma_c$, as $|\mathbf{x}|^{-2(1-a)}$, for $z=0$, and as $|z|^{-2(1-a)/(2-a)}$, for $\mathbf{x}=\mathbf{0}$, in contrast to 3D nematics where these correlations decay as $|\mathbf{x}|^{-1}$ and $|z|^{-1}$.

Thus, the relationship to the KPZ model provides an understanding of 3D smectics for the *particular* value of $\gamma_{sm} = \gamma_c = \pm (K_{sm} B_{sm})^{1/2}$, Eq. (5). For a *general* γ_{sm} , we analyzed Eq. (1) by a one-loop renormalization-group (RG) transformation. For $d=3$ (and for any $d \neq 2$), the RG flow pattern has a *separatrix* occurring for $|\gamma_{sm}| = |\gamma_c| = (K_{sm} B_{sm})^{1/2}$. For $|\gamma_{sm}| < |\gamma_c|$, flows iterate to the symmetric, ordinary smectic line with $\gamma_{sm}=0$. So, for $|\gamma_{sm}| < |\gamma_c|$, at the longest length scales, one has ordinary Landau-Peierls behavior [13] and logarithmic corrections of Grinstein and Pelcovits [1]. Along the separatrix $|\gamma_{sm}| = |\gamma_c|$, our RG actually reduces to that of KPZ [7], and one has the behavior discussed above [20]. The region $|\gamma_{sm}| > |\gamma_c|$ is a runaway region: the one-loop RG flows drive $|\gamma_{sm}|$ to ∞ . So, the KPZ separatrix $|\gamma_{sm}| = |\gamma_c| = (K_{sm} B_{sm})^{1/2}$ is, in fact, a *critical line* between a Landau-Peierls phase ($|\gamma_{sm}| < |\gamma_c|$) and an “unstable” region ($|\gamma_{sm}| > |\gamma_c|$). A preliminary mean-field investigation of Eq. (1) in the region $|\gamma_{sm}| > |\gamma_c|$ indicates the onset of an undulated phase, with $\langle u(\mathbf{x}, z) \rangle \neq 0$, in the form of a superposition of modulations with wave vectors normal to the z axis.

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Note added.—After this work was done, a related work, Ref. [21], was brought to our attention.

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- [16] The rotationally invariant KPZ model (2) is usually represented in a truncated form; only the terms up to the second order in $u(\mathbf{x}, z) = h(\mathbf{x}, z) - z$ are kept since higher-order terms are irrelevant [7]. The rotationally invariant smectic model (1) can be (with the same justification) truncated up to terms of the fourth order in u . The truncated models (the KPZ and the smectic one) preserve infinitesimal rotational invariance (known as "Galilean" invariance in the KPZ case). By applying the formalism of Refs. [15] to the truncated KPZ model, one gets the truncated smectic model as H_{eff} , while the Jacobian $J(h) = |D\eta/Dh|$ turns out to be a constant. In the case of the full nonlinear KPZ model (2), J is a functional of h , which, however, contributes only irrelevant terms not affecting the universal scaling.
- [17] The γ_{sm} term in (1) contributes, to the *lowest* order in u , the *cubic* term $(\nabla_{\mathbf{x}} u)^2 \nabla_z^2 u$. Thus, the quadratic, harmonic approximation to (1) is *insensitive* to the value of γ_{sm} , and the harmonic propagator is of the standard form $G_0 = (B_{\text{sm}} q_z^2 + K_{\text{sm}} q_x^4)^{-1}$. The renormalized propagator G_R is obtained by replacing $B_{\text{sm}} \rightarrow B_{\text{sm}}(q)$, $K_{\text{sm}} \rightarrow K_{\text{sm}}(q)$, and the form of the renormalized elastic constants is extracted by requiring G_R to yield the scaling of corrections as given by (6). This yields
- $$K_{\text{sm}} \sim q_x^{d-3+a} \sim q_z^{(d-3+a)/(2-a)},$$
- $$B_{\text{sm}} \sim q_x^{d-3+3a} \sim q_z^{(d-3+3a)/(2-a)}.$$
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- [20] For $d < 3$, our one-loop RG gives a *perturbative* anharmonic fixed point (A) with $\gamma_{\text{sm}}^* = 0$, yielding, in the region $|\gamma_{\text{sm}}| < |\gamma_c|$, the scaling as in Eq. (6), with $\alpha = 3(3-d)/(8-d)$, $\beta = 3(3-d)/(7+d)$. Scaling along the critical, KPZ line ($|\gamma_{\text{sm}}| = |\gamma_c|$) is regulated by a different, *nonperturbative* fixed point (KPZ) with $\gamma_{\text{sm}}^* \neq 0$. For $d > 2$, KPZ is unstable with respect to A . As $d \rightarrow 2$, this instability vanishes, and a *line* of fixed points containing both A and KPZ is formed in $d = 2$. The appearance of this line is related to the fact that, in 2D, the γ_{sm} term in (1) becomes a boundary term not affecting bulk fluctuations. So, all the points of the fixed line yield the *same* scaling behavior with $\alpha = 1/2$, $\beta = 1/3$ *exactly*. Note also that, in 2D, the above one-loop results for α and β for A agree with the exact values for KPZ. In 2D, the one-loop theory becomes exact in the continuum (see also Ref. [7]).
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